

NPS-CS-96-003

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## A Note on Typing Variables and References

by

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February 1996

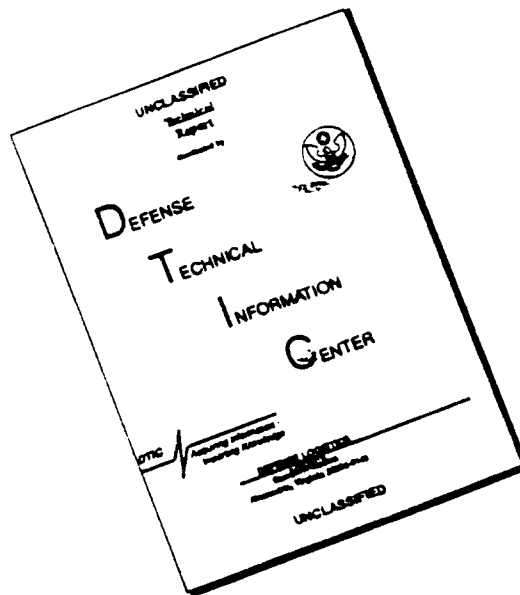
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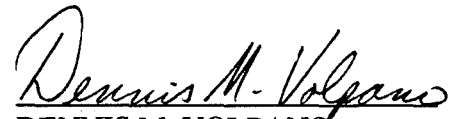
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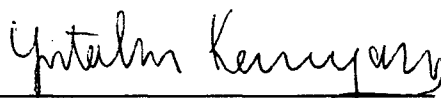
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This report was prepared as part of the Advanced Type Systems Project at the Naval Postgraduate School, which is currently funded by the National Science Foundation under Agreement No. CCR-9400592 and CCR-9414421. Any opinions, findings, and conclusions or recommendations expressed in this report are those of the author and do not necessarily reflect the views of the National Science Foundation.


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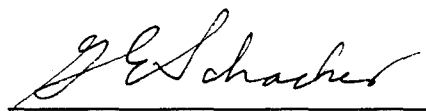
  
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## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NPSCS-96 -003		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION Computer Science Dept. Naval Postgraduate School	6b. OFFICE SYMBOL (if applicable) CS	7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943		7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION National Science Foundation	8b. OFFICE SYMBOL (if applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code) 4201 Wilson Blvd Arlington, VA 22230		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO.	PROJECT NO.
		TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) A Note on Typing Variables and References			
12. PERSONAL AUTHOR(S) Dennis Volpano and Geoffrey Smith			
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM _____ TO _____	14. DATE OF REPORT (Year, Month, Day) February 8, 1996	15. PAGE COUNT 7
16. SUPPLEMENTARY NOTATION			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)  polymorphic types, references, variables, and weak types	
FIELD	GROUP		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)  We consider the polymorphic typing of variables and references with C's address-of operator '&' in the context of nonweak types. A natural semantics and type system are given for a polymorphically-typed imperative language with first class functions. The type system is proved sound with respect to the natural semantics.			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dennis Volpano, Assistant Professor		22b. TELEPHONE (Include Area Code) (408) 656-3091	22c. OFFICE SYMBOL CS/ Vo

# A Note on Typing Variables and References

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February 8, 1996

Polymorphic typing of variables and references is considered in [1]. However, a treatment of the address-of operator '&' in the context of nonweak types is not given. The operator is treated in [2] but only in the context of weak types, since every type in that system is weak. In this note, the semantics and subject reduction theorem of [1] are reformulated in order to accomodate '&' in the presence of nonweak types.

The syntax of the language in [1] is extended as follows:

(Expressions)  $e ::= \&e \mid l.1$   
(Values)  $v ::= l.0$

Meta-variable  $l$  ranges over locations. We say  $l.1$  is a *variable* and  $l.0$  is a *reference*. Unlike references, variables are not values. Variables and references replace variable locations and reference locations respectively in the syntax of [1].

Typing rules (REFLOC) and (VARLOC) of [1] are changed and a typing rule for '&' is added—see Figure 1. The domain of a location typing is no longer partitioned into variable and reference locations.

Some changes are needed in the evaluation rules. These changes are reflected in the new rules given in Figure 2.

We now turn to subject reduction. First, we introduce some lemmas:

**Lemma 1** (Superfluosness) *If  $\lambda; \gamma \vdash e : \tau$  and  $l \notin \text{dom}(\lambda)$ , then  $\lambda[l : \tau']; \gamma \vdash e : \tau$ .*

**Lemma 2** (Substitution) *If  $\lambda; \gamma \vdash v : \sigma$  and  $\lambda; \gamma[x : \sigma] \vdash e : \tau$ , then  $\lambda; \gamma \vdash [v/x]e : \tau$ . Also, if  $\lambda; \gamma \vdash l.1 : \tau \text{ var}$  and  $\lambda; \gamma[x : \tau \text{ var}] \vdash e : \tau'$ , then  $\lambda; \gamma \vdash [l.1/x]e : \tau'$ .*

$$\begin{array}{ll} \text{(REFLOC)} & \lambda; \gamma \vdash l.0 : \tau \text{ ref} \quad \lambda(l) = \tau \\ \text{(VARLOC)} & \lambda; \gamma \vdash l.1 : \tau \text{ var} \quad \lambda(l) = \tau \\ \text{(ADDRESS)} & \frac{\lambda; \gamma \vdash e : \tau \text{ var}, \quad \tau \text{ is weak}}{\lambda; \gamma \vdash \&e : \tau \text{ ref}} \end{array}$$

Figure 1: New Rules of the Type System

(CONTENTS)	$\mu \vdash l.1 \Rightarrow \mu(l), \mu$
(BINDVAR)	$\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ l \notin \text{dom}(\mu_1) \\ \mu_1[l := v_1] \vdash [l.1/x]e_2 \Rightarrow v_2, \mu_2 \end{array}}{\mu \vdash \text{letvar } x := e_1 \text{ in } e_2 \Rightarrow v_2, \mu_2}$
(UPDATE)	$\frac{\mu \vdash e \Rightarrow v, \mu'}{\mu \vdash l.1 := e \Rightarrow \text{unit}, \mu'[l := v]}$ $\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow l.0, \mu_1 \\ \mu_1 \vdash e_2 \Rightarrow v, \mu_2 \end{array}}{\mu \vdash *e_1 := e_2 \Rightarrow \text{unit}, \mu_2[l := v]}$
(ADDROF)	$\mu \vdash \&l.1 \Rightarrow l.0, \mu$ $\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash \&*e \Rightarrow l.0, \mu'}$
(ALLOC)	$\frac{\begin{array}{l} \mu \vdash e \Rightarrow v, \mu' \\ l \notin \text{dom}(\mu') \end{array}}{\mu \vdash \text{ref } e \Rightarrow l.0, \mu'[l := v]}$
(DEREF)	$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash *e \Rightarrow \mu'(l), \mu'}$

Figure 2: The New Evaluation Rules

**Lemma 3** ( $\forall$ -intro) *If  $\lambda; \gamma \vdash e : \sigma$  and  $\alpha$  does not occur free in  $\lambda$  or in  $\gamma$ , then  $\lambda; \gamma \vdash e : \forall \alpha. \sigma$ .*

**Lemma 4** *If  $\lambda[l : \tau]; \gamma \vdash e : \tau'$  and  $l$  does not occur in  $e$ , then  $\lambda; \gamma \vdash e : \tau'$ .*

The preceding lemmas are straightforward variants of those in [1].

The subject reduction theorem now becomes:

**Theorem 5** *Suppose that  $\mu \vdash e \Rightarrow v, \mu'$ ,  $\lambda \vdash e : \tau$ ,  $\mu : \lambda$ , and  $\lambda(l)$  is weak if  $l.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e$ , or  $l.0$  occurs in the range of  $\mu$  or in  $e$ . Then there exists a  $\lambda'$  such that  $\lambda \subseteq \lambda'$ ,  $\mu' : \lambda'$ ,  $\lambda' \vdash v : \tau$ , and  $\lambda'(l)$  is weak if  $l.1$  or  $l.0$  occurs in the range of  $\mu'$  or in  $v$ .*

*Proof.* The proof is by induction on the structure of the derivation of  $\mu \vdash e \Rightarrow v, \mu'$ .

For brevity, we present only the interesting cases: (BIND), when  $e_1$  is not a value, and the evaluation rules of Figure 2.

(BIND). Suppose  $e_1$  is not a value. Then the evaluation must end with

$$\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ \mu_1 \vdash [v_1/x]e_2 \Rightarrow v_2, \mu_2 \end{array}}{\mu \vdash \text{let } x = e_1 \text{ in } e_2 \Rightarrow v_2, \mu_2}$$

while the typing must end with

$$\frac{\begin{array}{l} \lambda \vdash e_1 : \tau_1 \\ \lambda; [x : \text{AppClose}_\lambda(\tau_1)] \vdash e_2 : \tau_2 \end{array}}{\lambda \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2}$$

Also,  $\mu : \lambda$  and  $\lambda(l)$  is weak if either  $l.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e_1$  or  $e_2$ , or  $l.0$  occurs in the range of  $\mu$  or in  $e_1$  or  $e_2$ .

By induction, there exists a  $\lambda_1$  such that  $\lambda \subseteq \lambda_1$ ,  $\mu_1 : \lambda_1$ ,  $\lambda_1 \vdash v_1 : \tau_1$ , and  $\lambda_1(l)$  is weak if  $l.1$  or  $l.0$  occurs in the range of  $\mu_1$  or in  $v_1$ .

Now to apply induction again we want to show that

$$\lambda_1 \vdash [v_1/x]e_2 : \tau_2.$$

By Lemma 1 we have

$$\lambda_1; [x : \text{AppClose}_\lambda(\tau_1)] \vdash e_2 : \tau_2,$$

so we can apply Lemma 2 to get what we want provided that we can show

$$\lambda_1 \vdash v_1 : \text{AppClose}_\lambda(\tau_1).$$

Now, applying Lemma 3 to  $\lambda_1 \vdash v_1 : \tau_1$  we can get  $\lambda_1 \vdash v_1 : \text{AppClose}_{\lambda_1}(\tau_1)$ , but this is not good enough, because  $\lambda_1$  may contain free strong type variables that are not free in  $\lambda$ . To proceed, we exploit our knowledge about what locations can occur in  $v_1$ .

Let  $\lambda_1^-$  be formed by removing from  $\lambda_1$  any typings  $l : \tau$  such that  $\tau$  is not weak. By the above use of induction, this process does not remove any typings of locations that occur in  $v_1$ , as all such locations have weak types. So by Lemma 4,  $\lambda_1^- \vdash v_1 : \tau_1$ . Hence, by Lemma 3,  $\lambda_1^- \vdash v_1 : \text{AppClose}_\lambda(\tau_1)$ , since  $\lambda_1^-$  contains no strong type

variables. Lemma 1 then gives  $\lambda_1 \vdash v_1 : AppClose_\lambda(\tau_1)$ , and finally by Lemma 2 we get  $\lambda_1 \vdash [v_1/x]e_2 : \tau_2$ .

By the use of induction above,  $\lambda_1(l)$  is weak if  $l.1$  or  $l.0$  occurs in the range of  $\mu_1$ . If a variable  $l.1$  occurs in a  $\lambda$ -abstraction in  $[v_1/x]e_2$ , then either it occurs in  $v_1$  or in a  $\lambda$ -abstraction in  $e_2$ . In the first case,  $\lambda_1(l)$  is weak by the above use of induction; in the second case,  $\lambda(l)$  is weak by the hypothesis, and so  $\lambda_1(l)$  is weak since  $\lambda \subseteq \lambda_1$ . Furthermore, if a reference  $l.0$  occurs in  $[v_1/x]e_2$ , then either it occurs in  $v_1$  or  $e_2$ . In the former case,  $\lambda_1(l)$  is weak by the above use of induction, and in the latter,  $\lambda(l)$  is weak by the hypothesis, and so  $\lambda_1(l)$  is weak.

Hence we can use induction a second time to show that there exists a  $\lambda'$  such that  $\lambda_1 \subseteq \lambda'$ ,  $\mu_2 : \lambda'$ ,  $\lambda' \vdash v_2 : \tau_2$ , and  $\lambda'(l)$  is weak if  $l.1$  or  $l.0$  occurs in the range of  $\mu_2$  or in  $v_2$ . Since  $\lambda \subseteq \lambda_1 \subseteq \lambda'$ , we are done.

(BINDVAR). The evaluation must end with

$$\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ l \notin dom(\mu_1) \\ \mu_1[l := v_1] \vdash [l.1/x]e_2 \Rightarrow v_2, \mu_2 \end{array}}{\mu \vdash \text{letvar } x := e_1 \text{ in } e_2 \Rightarrow v_2, \mu_2}$$

while the typing must end with

$$\frac{\begin{array}{l} \lambda \vdash e_1 : \tau_1 \\ \lambda; [x : \tau_1 \text{ var}] \vdash e_2 : \tau_2 \\ \text{If } x \text{ occurs in a } \lambda\text{-abstraction in } e_2 \text{ then } \tau_1 \text{ is weak.} \end{array}}{\lambda \vdash \text{letvar } x := e_1 \text{ in } e_2 : \tau_2}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if either  $l'.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e_1$  or  $e_2$ , or  $l'.0$  occurs in the range of  $\mu$  or in  $e_1$  or  $e_2$ .

By induction, there exists a  $\lambda_1$  such that  $\lambda \subseteq \lambda_1$ ,  $\mu_1 : \lambda_1$ ,  $\lambda_1 \vdash v_1 : \tau_1$ , and  $\lambda_1(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu_1$  or in  $v_1$ .

Since  $l \notin dom(\lambda_1)$ ,  $\lambda_1 \subseteq \lambda_1[l : \tau_1]$ .

Since  $\lambda_1[l : \tau_1] \vdash l.1 : \tau_1 \text{ var}$  and (by Lemma 1)  $\lambda_1[l : \tau_1]; [x : \tau_1 \text{ var}] \vdash e_2 : \tau_2$ , we can apply Lemma 2 to get

$$\lambda_1[l : \tau_1] \vdash [l.1/x]e_2 : \tau_2$$

Also,  $\mu_1[l := v_1] : \lambda_1[l : \tau_1]$  by Lemma 1.

Next, by the use of induction above,  $\lambda_1(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu_1[l := v_1]$ . Thus,  $\lambda_1[l : \tau_1](l')$  is weak since  $\lambda_1 \subseteq \lambda_1[l : \tau_1]$ . Now suppose that a variable  $l'.1$  occurs in a  $\lambda$ -abstraction in  $[l.1/x]e_2$ . Then either  $l'.1$  occurs in a  $\lambda$ -abstraction in  $e_2$ , or else  $l' = l$  and  $x$  occurs in a  $\lambda$ -abstraction in  $e_2$ . In the first case, by the hypothesis,  $\lambda(l')$  is weak and so  $\lambda_1[l : \tau_1](l')$  is weak. In the second case, by the restriction on the (LETVAR) rule,  $\tau_1$  is weak, and so  $\lambda_1[l : \tau_1](l')$  is weak. Finally, if  $l'.0$  occurs in  $[l.1/x]e_2$  then it occurs in  $e_2$ . Thus, by the hypothesis,  $\lambda(l')$  is weak and so  $\lambda_1[l : \tau_1](l')$  is weak.

So by a second use of induction, there exists a  $\lambda'$  such that  $\lambda_1[l : \tau_1] \subseteq \lambda'$ ,  $\mu_2 : \lambda'$ ,  $\lambda' \vdash v_2 : \tau_2$ , and  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu_2$  or in  $v_2$ . Since  $\lambda \subseteq \lambda_1 \subseteq \lambda_1[l : \tau_1] \subseteq \lambda'$ , we are done.

(ADDROF). Suppose the evaluation ends with

$$\mu \vdash l.1 \Rightarrow l.0, \mu$$



while the typing ends with

$$\frac{\lambda \vdash l.1 : \tau \text{ var}, \tau \text{ is weak}}{\lambda \vdash \&l.1 : \tau \text{ ref}}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if either  $l'.1$  or  $l'.0$  occurs in the range of  $\mu$ . Since  $\lambda \vdash l.1 : \tau \text{ var}$ , we have  $\lambda(l) = \tau$  by rule (VARLOC). Thus,  $\lambda \vdash l.0 : \tau \text{ ref}$  by (REFLOC). Furthermore, by the restriction on rule (ADDRESS),  $\tau$ , or  $\lambda(l)$ , is weak.

Now suppose the evaluation ends with

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash \&*e \Rightarrow l.0, \mu'}$$

while the typing ends with

$$\frac{\lambda \vdash e : \tau \text{ ref}}{\frac{\lambda \vdash *e : \tau \text{ var}, \tau \text{ is weak}}{\lambda \vdash \&*e : \tau \text{ ref}}}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if  $l'.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e$ , or  $l'.0$  occurs in the range of  $\mu$  or in  $e$ .

By induction, there is a  $\lambda'$  such that  $\lambda \subseteq \lambda'$ ,  $\mu' : \lambda'$ ,  $\lambda' \vdash l.0 : \tau \text{ ref}$ ,  $\lambda'(l)$  is weak, and  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu'$ . And, we're done.

(CONTENTS). The evaluation must end with

$$\mu \vdash l.1 \Rightarrow \mu(l), \mu$$

while the typing must end with

$$\frac{\lambda \vdash l.1 : \tau \text{ var}}{\lambda \vdash l.1 : \tau}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if either  $l'.1$  or  $l'.0$  occurs in the range of  $\mu$ . From  $\mu : \lambda$ , we have  $\lambda \vdash \mu(l) : \lambda(l)$ . Since  $\lambda \vdash l.1 : \tau \text{ var}$ , we have  $\lambda(l) = \tau$ , so  $\lambda \vdash \mu(l) : \tau$ .

(UPDATE). Suppose the evaluation ends with

$$\frac{\mu \vdash e \Rightarrow v, \mu'}{\mu \vdash l.1 := e \Rightarrow \text{unit}, \mu'[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash l.1 : \tau \text{ var} \quad \lambda \vdash e : \tau}{\lambda \vdash l.1 := e : \text{unit}}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if  $l'.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e$ , or  $l'.0$  occurs in the range of  $\mu$  or in  $e$ .

By induction, there exists a  $\lambda'$  such that  $\lambda \subseteq \lambda'$ ,  $\mu' : \lambda'$ ,  $\lambda' \vdash v : \tau$ , and  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu'$  or in  $e$ .

By rule (LIT),  $\lambda' \vdash \text{unit} : \text{unit}$ . Since  $\lambda \vdash l.1 : \tau \text{ var}$ ,  $\lambda(l) = \tau$  by (VARLOC). So  $l \in \text{dom}(\lambda')$  since  $\lambda \subseteq \lambda'$ , and thus  $\text{dom}(\mu'[l := v]) = \text{dom}(\lambda')$ . If  $l'$  is a location such

that  $l' \neq l$ , then  $\lambda' \vdash \mu'(l') : \lambda'(l')$  since  $\mu' : \lambda'$ . If  $l' = l$  then  $\mu'[l := v](l') = v$ . So  $\lambda' \vdash \mu'[l := v](l') : \tau$  since  $\lambda' \vdash v : \tau$ . Thus,  $\mu'[l := v] : \lambda'$ . Finally, by the above use of induction,  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu'[l := v]$ .

Now suppose the evaluation ends with

$$\frac{\mu \vdash e_1 \Rightarrow l.0, \mu_1 \quad \mu_1 \vdash e_2 \Rightarrow v, \mu_2}{\mu \vdash *e_1 := e_2 \Rightarrow \mathbf{unit}, \mu_2[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash *e_1 : \tau \text{ var} \quad \lambda \vdash e_2 : \tau}{\lambda \vdash *e_1 := e_2 : \mathbf{unit}}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if  $l'.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e_1$  or  $e_2$ , or  $l'.0$  occurs in the range of  $\mu$  or in  $e_1$  or  $e_2$ .

By rule (L-VAL),  $\lambda \vdash e_1 : \tau \text{ ref}$ . By induction, there exists a  $\lambda_1$  such that  $\lambda \subseteq \lambda_1$ ,  $\mu_1 : \lambda_1$ ,  $\lambda_1 \vdash l.0 : \tau \text{ ref}$ ,  $\lambda_1(l)$  is weak, and  $\lambda_1(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu_1$ . By Lemma 1,  $\lambda_1 \vdash e_2 : \tau$ . Suppose that a variable  $l'.1$  occurs in a  $\lambda$ -abstraction in  $e_2$ . Then by the hypothesis,  $\lambda(l')$  is weak and so is  $\lambda_1(l')$  since  $\lambda \subseteq \lambda_1$ . Likewise, if  $l'.0$  occurs in  $e_2$ , then  $\lambda(l')$  is weak and thus so is  $\lambda_1(l')$ .

So by a second use of induction, there is a  $\lambda'$  such that  $\lambda_1 \subseteq \lambda'$ ,  $\mu_2 : \lambda'$ ,  $\lambda' \vdash v : \tau$ , and  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu_2$  or in  $v$ . The proof is now similar to the first (UPDATE) case above.

(ALLOC). The evaluation must end with

$$\frac{\mu \vdash e \Rightarrow v, \mu' \quad l \notin \text{dom}(\mu')}{\mu \vdash \mathbf{ref} e \Rightarrow l.0, \mu'[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash e : \tau, \tau \text{ is weak}}{\lambda \vdash \mathbf{ref} e : \tau \text{ ref}}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if  $l'.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e$ , or  $l'.0$  occurs in the range of  $\mu$  or in  $e$ .

By induction, there exists a  $\lambda'$  such that  $\lambda \subseteq \lambda'$ ,  $\mu' : \lambda'$ ,  $\lambda' \vdash v : \tau$ , and  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu'$  or in  $v$ .

Now  $\lambda' \subseteq \lambda'[l : \tau]$  since  $l \notin \text{dom}(\mu')$ .

By Lemma 1 and the above use of induction,  $\mu'[l := v] : \lambda'[l : \tau]$ . Furthermore,  $\lambda'[l : \tau] \vdash l.0 : \tau \text{ ref}$  by rule (REFLOC). Again by the above use of induction,  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu'[l := v]$ , and hence  $\lambda'[l : \tau](l')$  is weak since  $\lambda' \subseteq \lambda'[l : \tau]$ . Finally,  $\lambda'[l : \tau](l) = \tau$  and  $\tau$  is weak by the restriction on rule (REF).

(DEREF). The evaluation must end with

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash *e \Rightarrow \mu'(l), \mu'}$$

while the typing ends with

$$\frac{\frac{\lambda \vdash e : \tau \text{ ref}}{\lambda \vdash *e : \tau \text{ var}}}{\lambda \vdash *e : \tau}$$

Also,  $\mu : \lambda$  and  $\lambda(l')$  is weak if  $l'.1$  occurs in the range of  $\mu$  or in a  $\lambda$ -abstraction in  $e$ , or  $l'.0$  occurs in the range of  $\mu$  or in  $e$ .

By induction, there exists a  $\lambda'$  such that  $\lambda \subseteq \lambda'$ ,  $\mu' : \lambda'$ ,  $\lambda' \vdash l.0 : \tau \text{ ref}$ ,  $\lambda'(l)$  is weak, and  $\lambda'(l')$  is weak if  $l'.1$  or  $l'.0$  occurs in the range of  $\mu'$ .

Since  $\lambda' \vdash l.0 : \tau \text{ ref}$ ,  $\lambda'(l) = \tau$  by rule (REFLOC). Now  $\lambda' \vdash \mu'(l) : \lambda'(l)$ , since  $\mu' : \lambda'$ , so  $\lambda' \vdash \mu'(l) : \tau$ .  $\square$

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